Τρίτη 4 Ιουνίου

2η, εάν υπάρχει η διατήρησις

η λύση είναι ημιπεριοδική

$$x_0(t) = \sum_n \mathrm{Re}\left(a_n e^{i\omega_n t}\right) + \dots$$

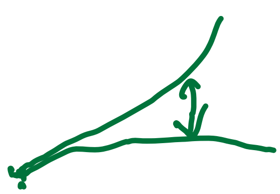
αρμονικές των ω_n

ολοκληρώσιμα συστήματα.

Χαμιλτονιανά

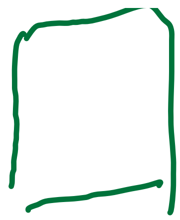
Poincaré 3 σώματα

λύσεις "χαοτικές" και απεριοδικές λύσεις (απεριοδικές)



Birkhoff ~1930

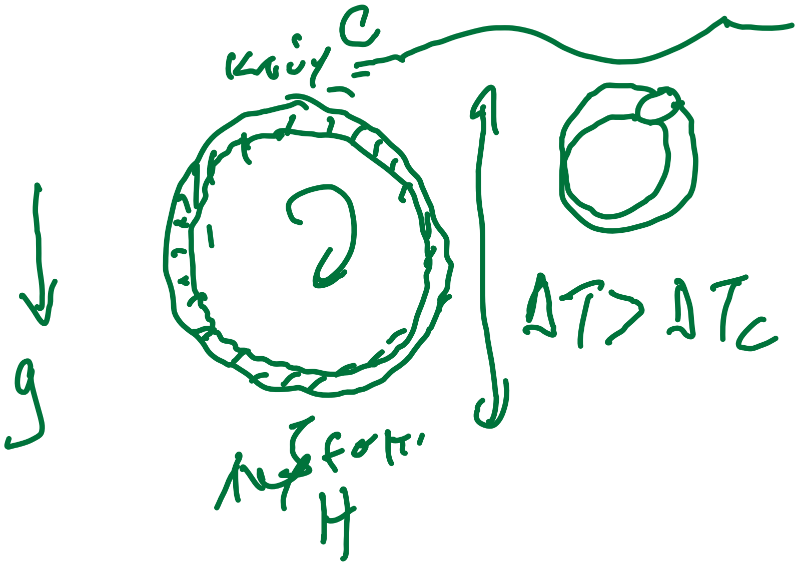
$\boxed{\text{Boltzmann}}$

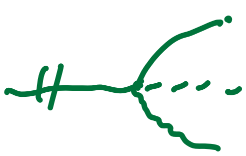


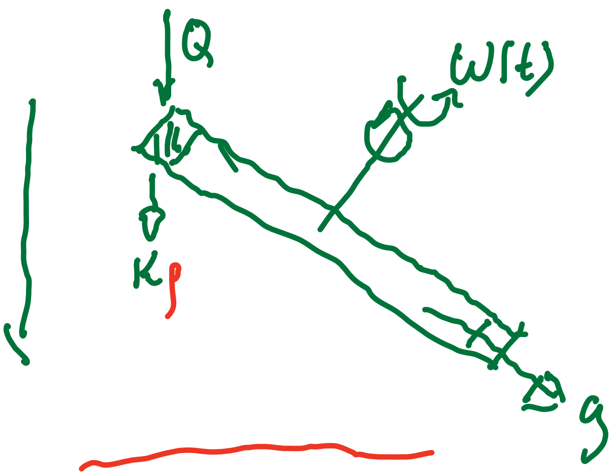
Kolmogorov – Arnold

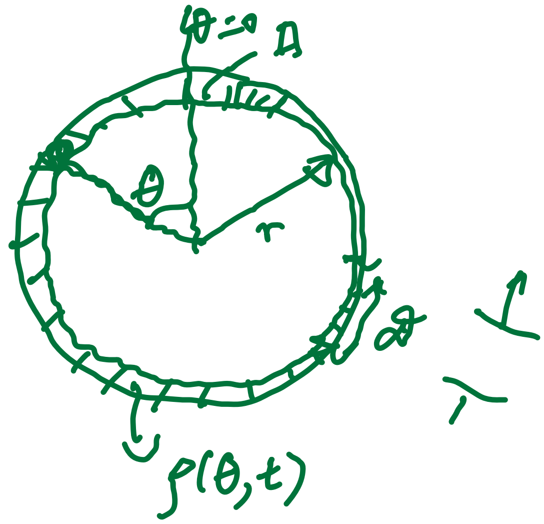
Ανάλυση τυρβώδη ροή. ν
θερμότητα κ

$\boxed{\text{Lorenz} \sim 1963}$









$$M = \int_\Omega \rho\, dV = \int_\Omega \rho A r\, d\theta$$

$$\frac{d}{dt} M = -\int_{\partial\Omega} \rho\, \vec{u}\cdot\hat{n}\, dS + \int Q\, dV - K\int \rho\, dV$$

$$\frac{d}{dt}\int_\Omega \rho A r\, d\theta = -\int \nabla\cdot(\rho\vec{u})\, dV + \int Q\, dV - K\int \rho\, dV$$

$$\nabla\cdot(\rho\vec{u}) = \frac{1}{r}\frac{d}{d\theta}(\rho\omega r) = \omega\frac{\partial\rho}{\partial\theta}$$

$$\frac{d}{dt}\int_\Omega \rho\, dV = \int_\Omega \left(-\omega\frac{\partial\rho}{\partial\theta} + Q - K\rho\right) dV$$

$$\int_{\partial}\left(\frac{\partial\rho}{\partial t}+w\frac{\partial\rho}{\partial\theta}-Q+k\rho\right)dV=0$$

$$\boxed{\frac{\partial\rho}{\partial t}=-w\frac{\partial\rho}{\partial\theta}+Q-k\rho}$$

$$I=I_0+\int\rho r^2(\underbrace{Ar\,d\theta}_{dV})$$

$$=I_0+Ar^3\int_0^{2\pi}\rho\,d\theta=I(t)$$

$$-\int\rho\vec{u}\cdot\vec{n}\,ds$$

ds

$$-\int\rho\vec{u}\cdot\vec{n}\,ds$$

[illegible] ότι ∂ είναι [illegible]

$$\frac{d}{dt}(I\omega)=-\nu w+\int_0^{2\pi}gAr^2\rho\sin\theta\,d\theta$$

1 $r\sin\theta$

2π

$$I = I_0 + A r^3 \int_0^{} \rho \, d\theta$$

$t \to \infty$

$\omega(t)$

$\rho(\theta, t)$

$$\rightarrow \frac{\partial \rho}{\partial t} = -\omega \frac{\partial \rho}{\partial \theta} + Q^{(\theta)} - K\rho$$

$Q(\theta) = Q(-\theta)$

$$Q(\theta) = \sum_{n=0}^{\infty} q_n \cos(n\theta)$$

$$\rho(\theta, t) = \sum_{n=0}^{\infty} \alpha_n(t) \sin(n\theta) + \beta_n(t) \cos(n\theta)$$

$$\int_0^{2\pi} \rho \sin\theta \, d\theta = \alpha_1 \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= \pi a_1$$

$$\boxed{\frac{d}{dt}(I\omega) = -\nu\omega + gA\pi r^2 a_1^{(t)}}$$

$$\int_0^{2\pi} \sin(n\theta) \sin(m\theta) = \pi \delta_{nm}$$

cos cos

$$\int_0^{2\pi} \sin n\theta \cos m\theta = 0$$

$$\int_0^{2\pi} \rho \, d\theta = 2\pi \beta_0(t)$$

$$\boxed{I = I_0 + 2\pi A r^3 \beta_0^{(t)}}$$

$$\sum_0 \dot{\alpha}_n(t)\, \underline{S_n} + \dot{\beta}_n C_n = -\omega \sum_0 (n\, a_n C_n - n\, \beta_n S_n)$$

$$+ \sum_0^\infty q_n C_n - \kappa \sum_0 (\alpha_n S_n + \beta_n C_n)$$

$$\Updownarrow$$

$$\dot{\alpha}_n = n\omega\beta_n - \kappa\alpha_n$$

$$\dot{\beta}_n = -n\omega\alpha_n + q_n - \kappa\beta_n$$

$\alpha_n(0) =$

$\beta_n(0) = 0$

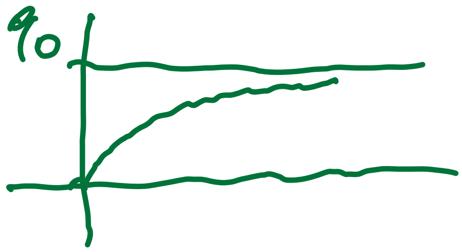
$\alpha_0 = 0$

$\underline{n=0}$ $$\boxed{\dot{\beta}_0 = q_0 - \kappa\beta_0}$$

$\underline{\kappa > 0,\ \nu \geq 0}$

$t \Rightarrow \infty$ $\quad \beta_0(\infty) = q_0/\kappa$

$$\beta_0(t) = \frac{q_0}{\underline{\kappa}}\left(1 - e^{-\kappa t}\right)$$



$$I \Rightarrow I_0 + 2\pi A r^3 \frac{q_0}{\kappa}$$

$t \gg 1/\kappa$

$$\boxed{\begin{aligned} \dot{\alpha}_1 &= \omega\beta_1 - \kappa\alpha_1 \\ \dot{\beta}_1 &= -\omega a_1 + q_1 - \kappa\beta_1 \\ \frac{d}{dt}(I\omega) &= -\nu\omega + g A \pi r^2 a_1 \end{aligned}}$$

f{1oi Lorenz}

$n > 1$

$$\boxed{\begin{aligned} \dot{\alpha}_n &= n\omega\beta_n - \kappa\alpha_n \\ \dot{\beta}_n &= -n\omega\alpha_n + q_n - \kappa\beta_n \end{aligned}} \longleftarrow \quad \omega(t)$$

να αποδείξω
αν φρ. λύσεις.

$$V = \frac{1}{2}\left(\alpha_n^2 + \beta_n^2\right) > 0$$

$$\dot{V} = \alpha_n\dot{\alpha}_n + \beta_n\dot{\beta}_n$$

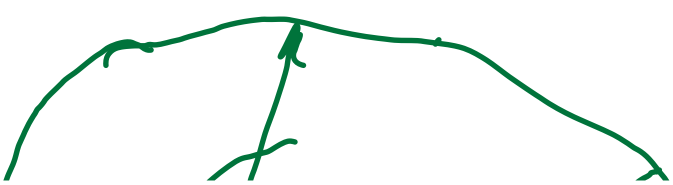
$$= \alpha_n\left(\cancel{n\omega\beta_n} - \kappa\alpha_n\right) + \beta_n\left(-\cancel{n\omega\alpha_n} + q_n - \kappa\beta_n\right)$$

$$= -\kappa\alpha_n^2 - \kappa\beta_n^2 + q_n\beta_n$$

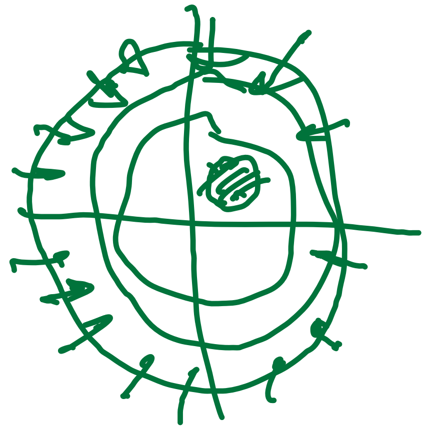
$$= -\kappa\left[\alpha_n^2 + \beta_n^2 - \frac{q_n\beta_n}{\kappa}\right]$$

$$= -\kappa\left[\alpha_n^2 + \left(\beta_n - \frac{q_n}{2\kappa}\right)^2 - \frac{q_n^2}{4\kappa^2}\right]$$

$$\dot{V} = -\kappa\left[\alpha_n^2 + \left(\beta_n - \frac{q_n}{2\kappa}\right)^2\right] + \frac{\kappa q_n^2}{4\kappa^2}$$

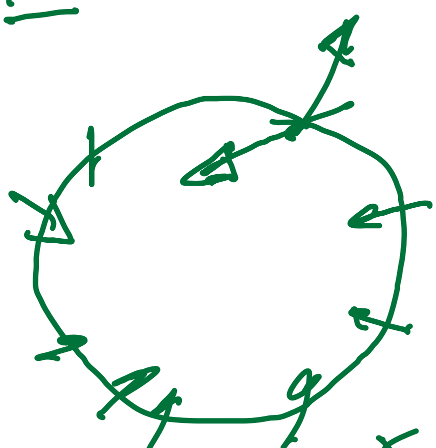


$$\alpha_n^2 + \beta_n^2$$



$\Rightarrow$ φραγμένες

$$\frac{dV}{dt} = \vec{\nabla} V \cdot \dot{x} < 0 \qquad \dot{x} = (\dot{\alpha}_4, \dot{\beta}_4)$$



η κίνηση θα είναι φραγμένη.

Εάν $q_4 = 0$, $n > 1$, $\quad Q = q_0 + q_1 \cos\theta$

!

Τότε $\alpha_4, \beta_4 \to 0$

Τότε $\dot{V} = -k(\alpha_4^2 + \beta_4^2)$

$a_4 = 0 \quad r > 1$

$$\dot{\alpha}_4 = nw\beta_2 - \kappa\alpha_4$$

$$\dot{\beta}_4 = -nwa_3 - \kappa\beta_4$$

$$\dot{\alpha}_4 = \dot{\beta}_4 = 0$$

$$V = \frac{\alpha_4^2 + \beta_4^2}{2} \quad \text{Energ} \quad \underline{\text{Lyapunc}}$$

$$\lim_{t\to\infty} \alpha_4(t) \to 0$$

$$\beta_4(t) \to 0$$

$$w \to x \qquad r = \frac{\pi A g r^2 q_1}{\kappa^2 \nu} \leftarrow \text{Rayleigh}$$

$$a_1 \to y$$

$$\beta_1 \to z \qquad \sigma = \frac{\nu}{\kappa I_\alpha} \sim \underline{10} \quad \text{Prandtle}$$

$$w = \kappa x$$

$$a_1 = \frac{\kappa\nu y}{\pi A g r^2}$$

$$\beta_1 = -\frac{\kappa\nu}{\pi A g r^2} z + \frac{q_1}{\kappa}$$

$$t = \tau/\kappa$$

$$\dot{x} = \sigma(y - x) \quad (w)$$

$$\dot{y} = rx - y - xz \quad (a_1)$$

$$\dot{z} = -bz + xy \quad (\beta_1)$$

$\mathcal{I}$

$\boxed{\sigma = 10}$ $\boxed{b = 8/3}$

[illegible] [illegible] $\quad V = \int_{D} dx\,dy\,dz$

$$\frac{dV}{dt} = \int_{D} \left(\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} \right) dx\,dy\,dz$$

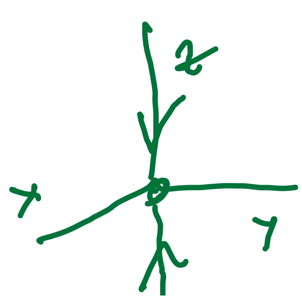
$$= \int_{D} (-\sigma - 1 - b)\, V$$

$$V(t) = e^{-(\sigma + 1 + b)t}\, V(0) \longrightarrow 0$$

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = rx - y - xz$$
$$\dot{z} = -bz + xy$$

$\underline{x = 0 = y = z}$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\sigma & \sigma & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$(\sigma + \lambda)(1 + \lambda) - r\sigma = 0$$

$$\lambda^2 + \lambda(\sigma + 1) + \sigma(1 - r) = 0$$

$-I$ $\qquad$ det

$\underline{\det > 0}$ $\qquad$ $\boxed{0 \le r < 1}$

<u>σταθερό</u> σημείο

<u>globally attractive</u>

$\underline{r<1}$ $\qquad (0,0,0)$

$$V = \frac{1}{2}\left(\frac{1}{\sigma}x^2 + y^2 + z^2\right)$$

$$\dot V = \frac{x\dot x}{\sigma} + y\dot y + z\dot z$$

$$= x\frac{\cancel{\sigma}(y-x)}{\cancel{\sigma}} + y(rx - y - \cancel{xz}) + z(-bz + \cancel{xy})$$

$$= xy - x^2 + rxy - y^2 - bz^2$$

$$= -\left(x^2 + y^2 - (r+1)xy\right) - bz^2$$

$$= -\left(\left(x - \frac{y(r+1)}{2}\right)^2 - \frac{y^2(r+1)^2}{4} + y^2\right) - bz^2$$

$$= -bz^2 - \left(x - \frac{y(r+1)}{2}\right)^2 - y^2\left(1 - \frac{(r+1)^2}{4}\right)$$

$\underline{\dot{V} < 0}$ εάν $1 - \frac{(r+1)^2}{4} > 0$

$2 > r+1$, $1 > r$!

$\underline{\text{Άρα}}$ είναι globally attracting.

$\underline{r=1}$

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x - y$$

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -\sigma & \sigma \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

δεν είναι διαγωνοποιήσιμο

το $(0,0,0)$ παραμένει $\overline{\text{globally}}$ $\underline{\text{attracting}}$.

$$\dot{x} = \sigma (y - x)$$
$$\dot{y} = rx - y - xz$$
$$\dot{z} = -bz + xy$$

[illegible]

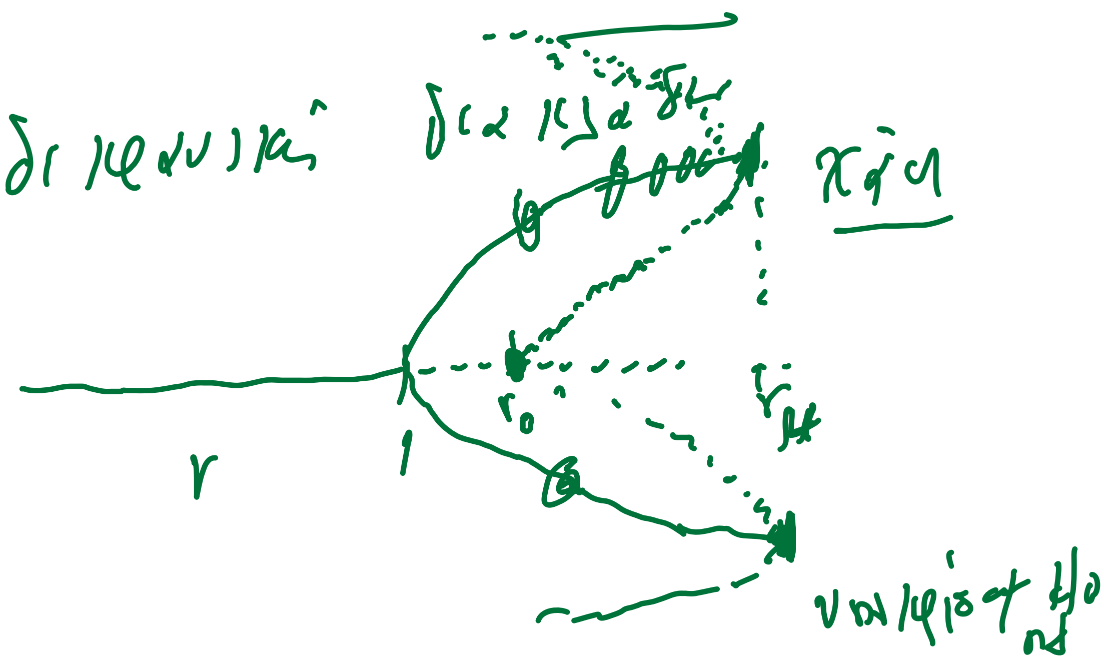
$$x_e = y_e$$
$$x_e z_e = (r-1) x_e$$

$x_e \neq 0$

$$z_e = (r-1)$$
$$x_e^2 = b(r-1)$$

$$z_e = r - 1 \ , \quad x_e = y_e = \pm\sqrt{b(r-1)}$$

$$r > 1$$



$\sigma = 10, \quad b = 8/3$

$r_H = 24.74, \quad r_0 = 13.926$

$\boxed{\text{συγχρονισμός}}$

$\dot{x} = \sigma(y - x)$	$\dot{x}_1 = \sigma(y_1 - x_1)$
$\dot{y} = r x - y - xz$	$\dot{y}_1 = r x - y_1 - x z_1$
$\dot{z} = -bz + xy$	$\dot{z}_1 = -b z_1 + x y_1$

$(x, y, z) \rightarrow$

$(x_1, y_1, z_1) \rightarrow (x, y, z)$

$e_1 = x - x_1 \qquad e_2 = y - y_1 \qquad e_3 = z - z_1$

$$V = \frac{1}{2}\left(\frac{e_1^2}{\sigma} + e_2^2 + e_3^2\right)$$

$$\dot e_1 = \sigma e_2 - \sigma e_1$$
$$\dot e_2 = -e_2 - x e_3$$
$$\dot e_3 = -b e_3 + x e_2$$

$$-e_1^2$$
$$\dot V = \underbrace{\sigma e_1 e_2}_{\sigma} - e_2^2 - x \cancel{e_2 e_3} - b e_3^2 + x \cancel{e_2 e_3}$$

$$= -e_2^2 - b e_3^2 + \underline{e_1 e_2} - e_1^2$$

$$= -\left(e_1 - \frac{e_2}{2}\right)^2 + \underbrace{\frac{e_2^2}{4} - e_2^2}_{-\frac{3e_2^2}{4}} - b e_3^2 < 0$$

άρα συγχρονίζονται
τα σήματα